

# The transverse polarized structure function in DIS and chiral symmetry breaking in QCD

Wei-Min Zhang and A. Harindranath\*

International Institute of Theoretical and Applied Physics

123 Office and laboratory, Iowa State University, Ames, Iowa 50011

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## Abstract

We study the polarized structure functions in QCD. We show that  $g_T$  which probes helicity flip interactions in hadrons on the light-front indeed measures the QCD dynamics of chiral symmetry breaking. The relation between chiral symmetry breaking and the observed  $g_2$  data is explored.

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\*On leave of absence from Saha Institute of Nuclear Physics, Sector 1, Block AF, Bidhan Nagar, Calcutta 700064 India.

Polarized structure functions, in particular, the transverse polarized structure function  $g_T = g_1 + g_2$ , have recently received much theoretical and experimental attention. Preliminary extraction of  $g_2$  has been made in the deeply inelastic scatterings (DIS) by the SMC experiment in CERN and the E143 experiment in SLAC very recently [1]. Unlike the longitudinal polarized structure function  $g_1$  which measures the quark helicity distribution in the longitudinal polarized hadrons, the physical interpretation of  $g_2$  is not simple [2]. Much theoretical work on  $g_2$  is currently concentrated on the questions whether  $g_2$  can still be described approximately by parton distributions [3], and whether it is a relatively good approximation to predict  $g_2$  from  $g_1$  via the Wandzura-Wilczek relation [4,5] or whether the quark-gluon coupling can provide a significant contribution to  $g_2$  [6].

In this letter we show that  $g_T$  probes the light-front helicity flip interactions in hadrons. The helicity flip on the light-front is the manifestation of chiral symmetry breaking in QCD. Therefore,  $g_T$  constitutes a direct measurement of QCD chiral symmetry breaking. We also explore the explicit relation between dynamical chiral symmetry breaking and the observed  $g_2$  data.

The polarized structure functions in DIS are defined from the antisymmetric part of hadronic tensor,

$$W_A^{\mu\nu} = -i\epsilon^{\mu\nu\lambda\sigma}q_\lambda \left\{ \frac{S_\sigma}{\nu}(g_1(x, Q^2) + g_2(x, Q^2)) - P_\sigma \frac{S \cdot q}{\nu^2} g_2(x, Q^2) \right\} \quad (1)$$

where  $P$  and  $S$  are the target four-momentum and polarization vector respectively ( $P^2 = M^2, S^2 = -M^2, S \cdot P = 0$ ), and  $q$  is the virtual-photon four momentum ( $Q^2 = -q^2, \nu = P \cdot q, x = \frac{Q^2}{2\nu}$ ). On the other hand, the hadronic tensor is related to the forward virtual-photon hadron Compton scattering amplitude:

$$W^{\mu\nu} = \frac{1}{4\pi} \text{Im} T^{\mu\nu} \quad , \quad T^{\mu\nu} = i \int d^4\xi e^{iq \cdot \xi} \langle PS | T(J^\mu(\xi) J^\nu(0)) | PS \rangle. \quad (2)$$

We first derive the hadronic matrix element expression for  $g_1$  and  $g_2$ . We shall not begin with the assumptions that have been used in the previous derivations, such as the zero quark mass and zero transverse quark momentum limits in the naive quark model, the free quark

field assumption in the impulse approximation, and even the factorization assumption in the collinear expansion approximation.

We begin with the  $\frac{1}{q^-}$  expansion of  $T^{\mu\nu}$  [7],

$$q^- T^{\mu\nu} = \int d\xi^- d^2\xi_\perp e^{iq\cdot\xi} \langle PS | [J^\mu(\xi), J^\nu(0)]_{\xi^+=0} | PS \rangle + O\left(\frac{1}{q^-}\right), \quad (3)$$

where  $q^- = q^0 - q^z$ . For large  $Q^2$  and  $\nu$  limits in DIS which correspond to large  $q^-$ , we ignore the contributions from terms of the order  $\frac{1}{q^-}$  in eq.(3). What remains is proportional to a light-front current commutator which can be computed directly from QCD (where QCD is quantized on the light-front time surface  $\xi^+ = \xi^0 + \xi^3 = 0$  with the light-front gauge  $A_a^+ = 0$  [8–10]). Then we can show

$$g_1(x, Q^2) = \frac{1}{4\pi S^+} \int_{-\infty}^{\infty} d\eta e^{-i\eta x} \langle PS | \psi_+^\dagger(\xi^-) \mathcal{Q}^2 \gamma_5 \psi_+(0) + h.c. | PS \rangle, \quad (4)$$

$$\begin{aligned} g_T(x, Q^2) &= \frac{1}{8\pi(S_\perp - \frac{P_\perp}{P^+} S^+)} \int_{-\infty}^{\infty} d\eta e^{-i\eta x} \langle PS | (O_m + O_{k_\perp} + O_g) + h.c. | PS \rangle \\ &= g_T^m(x, Q^2) + g_T^{k_\perp}(x, Q^2) + g_T^g(x, Q^2), \end{aligned} \quad (5)$$

where the parameter  $\eta \equiv \frac{1}{2}P^+\xi^-$  with  $\xi^-$  being the light-front longitudinal coordinate, and  $\mathcal{Q}$  the quark charge operator. We have also defined  $\psi_+ \equiv \frac{1}{2}\gamma^0\gamma^+\psi$  which is the light-front quark field, and  $g_T \equiv g_1 + g_2$ . The operators in eq.(5) are given as follows:

$$\begin{aligned} O_m &= m\psi_+^\dagger(\xi^-) \mathcal{Q}^2 \gamma_\perp \left( \frac{1}{i\overrightarrow{\partial}^+} - \frac{1}{i\overleftarrow{\partial}^+} \right) \gamma_5 \psi_+(0), \\ O_{k_\perp} &= -\psi_+^\dagger(\xi^-) \mathcal{Q}^2 \left( \gamma_\perp \frac{1}{\overrightarrow{\partial}^+} \overrightarrow{\partial}_\perp + \overleftarrow{\partial}_\perp \frac{1}{\overleftarrow{\partial}^+} \gamma_\perp + 2\frac{P_\perp}{P^+} \right) \gamma_5 \psi_+(0), \\ O_g &= g\psi_+^\dagger(\xi^-) \mathcal{Q}^2 \left( \mathcal{A}_\perp(\xi^-) \frac{1}{i\overleftarrow{\partial}^+} \gamma_\perp - \gamma_\perp \frac{1}{i\overrightarrow{\partial}^+} \mathcal{A}_\perp(0) \right) \gamma_5 \psi_+(0) \end{aligned} \quad (6)$$

and  $m$  and  $g$  are the quark mass and quark-gluon coupling constant in QCD, and  $A_\perp = A_\perp^a T_a$  the transverse gauge field.

Since we work in the light-front gauge, the operators in eqs.(6) are well-defined. Eqs.(4-5) are also the general expressions for the target being in any arbitrary frame  $\{P^\mu\}$ . By using the light-front decomposition,  $\psi = \psi_+ + \psi_-$ ,  $\psi_- = \frac{\gamma^0}{i\overrightarrow{\partial}^+} (\not{D}_\perp + m)\psi_+$ , we can formally rewrite eqs.(4-5) in familiar expressions,

$$g_1(x, Q^2) = \frac{1}{8\pi S^+} \int_{-\infty}^{\infty} d\eta e^{-i\eta x} \langle PS | \bar{\psi}(\xi^-) \mathcal{Q}^2 \gamma^+ \gamma_5 \psi(0) + h.c. | PS \rangle, \quad (7)$$

$$g_T(x, Q^2) = \frac{1}{8\pi(S_{\perp} - \frac{P_{\perp}}{P^+} S^+)} \int_{-\infty}^{\infty} d\eta e^{-i\eta x} \langle PS | \bar{\psi}(\xi^-) \mathcal{Q}^2 \left( \gamma_{\perp} - \frac{P_{\perp}}{P^+} \gamma^+ \right) \gamma_5 \psi(0) + h.c. | PS \rangle. \quad (8)$$

However, the physical picture is clearer in the expressions eqs.(4-5), where as we can see  $g_T$  contains explicitly the contributions associated with the quark mass, quark transverse momentum and quark-gluon coupling. Note that  $g_2$  cannot be directly computed in the physical basis. We can extract  $g_2$  from  $g_1$  and  $g_T$ , only the latter two structure functions can be directly calculated and experimentally measured in the longitudinal and transverse polarized targets,  $|P\lambda\rangle$  and  $|PS_{\perp}\rangle$ , respectively.

Also note that eqs.(4-5) are expressed in terms of equal *light-front* time matrix elements. It is most convenient to analyze these matrix elements in light-front Fock space expansion. The results are

$$g_1(x, Q^2) = \frac{1}{2} \sum_i e_i^2 \Delta q_i^L(x, Q^2) \quad (9)$$

$$g_T(x, Q^2) = \frac{1}{2xM} \sum_i e_i^2 \left\{ m_i \Delta q_i^T(x, Q^2) + \Delta \mathcal{K}_i(x, Q^2) + g \mathcal{T}_i^g(x, Q^2) \right\}, \quad (10)$$

where  $i$  is the flavor index, the notation  $\Delta A_i \equiv A_i^+ - A_i^- + \bar{A}_i^+ - \bar{A}_i^-$ ,

$$q_i^{L\pm}(x, Q^2) = \int \frac{d^2 k_{\perp}}{2(2\pi)^3} \langle P\lambda | b_i^{\dagger}(x, k_{\perp}, \pm\lambda) b_i(x, k_{\perp}, \pm\lambda) | P\lambda \rangle, \quad (11)$$

$$q_i^{T\pm}(x, Q^2) = \int \frac{d^2 k_{\perp}}{2(2\pi)^3} \langle PS^1 | b_i^{\dagger}(x, k_{\perp}, \pm s^1) b_i(x, k_{\perp}, \pm s^1) | PS^1 \rangle, \quad (12)$$

$$\mathcal{K}_i^{\pm}(x, Q^2) = \int \frac{d^2 k_{\perp}}{2(2\pi)^3} \kappa^1 \langle PS^1 | b_i^{\dagger}(x, k_{\perp}, \pm\lambda) b_i(x, k_{\perp}, \pm\lambda) | PS^1 \rangle, \quad (13)$$

and  $\bar{q}_i^{\pm}$  and  $\bar{\mathcal{K}}_i^{\pm}$  have similar form for antiquarks. In eqs.(11-13),  $\lambda$  is the light-front helicity (the eigenvalue of the Pauli matrix  $\sigma_z$ ). Without loss of generality, we have also taken the transverse polarization of the target in the  $x$ -direction:  $S^1$ , and  $\kappa^1 = k_{\perp}^1 - xP_{\perp}^1$  is the  $x$ -component of the relative transverse quark momentum, while  $\mathcal{T}_i^g$  has no simple expression.

With the light-front quantization being utilized [11], the physical interpretation of the above results becomes rather simple. The  $g_1$  is purely determined by the quark and antiquark helicity distribution  $\Delta q_i^L$ . The transverse polarized structure function  $g_T$  contains three

contributions, as we have mentioned. The contribution associated with quark mass  $g_T^m$  is proportional to the transverse polarized distribution  $\Delta q_i^T$ . Apparently, the contribution associated with transverse quark momentum  $g_T^{k_\perp}$  is proportional to  $\Delta \mathcal{K}_i$  which measures averages of the transverse momentum  $\kappa_\perp$  of quarks and antiquarks with helicity up and down in the transverse polarized target. Besides,  $g_T$  also includes the contribution  $g_T^g$  from the quark-gluon coupling, which is proportional to  $\mathcal{T}_g$  and describes dynamical processes of a parton emitting and absorbing a gluon. At this step, formally the later two contributions in  $g_T$  do not have a simple parton picture, and they are the most interesting quantities in the current study of  $g_2$ . It has been suggested that the contribution proportional to quark mass is small since the current quark mass is small. Therefore, the later two contributions,  $g_T^{k_\perp}$  and  $g_T^g$ , appear to be dominant in the transverse polarized structure function.

However, we find that, first of all, the main contributions from  $g_T^{k_\perp}$  and  $g_T^g$  have indeed the simple parton picture just as  $g_T^m$  but they do not manifest at the tree level of QCD. Secondly, the nontrivial dynamics determined by  $g_T$  comes from the dynamical chiral symmetry breaking of nonperturbative QCD. To clearly see what is the physical origin of such  $g_T^{k_\perp}$  and  $g_T^g$  contributions and how dynamical chiral symmetry breaking dominates the physics of  $g_T$ , we must have further knowledge on the target bound state. The target state with transverse polarization in the  $x$ -direction can be expressed as a combination of the helicity up and down states:  $|PS^x\rangle = \frac{1}{\sqrt{2}}(|P \uparrow\rangle \pm |P \downarrow\rangle)$  for  $S^x = \pm M$ . Then we have

$$g_T(x, Q^2) = \frac{1}{8\pi M} \int_{-\infty}^{\infty} d\eta e^{-i\eta x} \frac{1}{2} \sum_{\lambda} \langle P\lambda | (O_m + O_{k_\perp} + O_g) + h.c. | P-\lambda \rangle. \quad (14)$$

This shows that  $g_T$  measures the helicity flip dynamics of hadrons.

So far, we have not specified the general structure of  $|PS\rangle$ . Generally, on the light-front,

$$|PS\rangle = \sum_{n, \lambda_i} \int' \frac{dx_i d^2 \kappa_{\perp i}}{2(2\pi)^3} |n, x_i P^+, x_i P_\perp + \kappa_{\perp i}, \lambda_i\rangle \Psi_n^S(x_i, \kappa_{\perp i}, \lambda_i), \quad (15)$$

where  $|n, x_i P^+, x_i P_\perp + \kappa_{\perp i}, \lambda_i\rangle$  is a Fock state with  $n$  constituents,  $\int'$  denotes the integral over the space  $(x_i, \kappa_{\perp i})$  with  $\sum_i x_i = 1$  and  $\sum_i \kappa_{\perp i} = 0$ , where  $x_i = \frac{k_i^+}{P^+}$ ,  $\kappa_{\perp i} = k_{\perp i} - x_i P_\perp$ , and  $k_i^+, k_{\perp i}$  are the longitudinal and transverse momentum of the  $i$ -th constituent with

helicity  $\lambda_i$ . The amplitude  $\Psi_n^S(x_i, \kappa_{\perp i}, \lambda_i)$  is determined by the QCD eigenvalue equation  $H_{QCD}^{LF}|PS\rangle = \frac{P_{\perp}^2 + M^2}{P^+}|PS\rangle$  which can be explicitly written as [9]

$$\left(M^2 - \sum_i \frac{\kappa_{i\perp}^2 + m_i^2}{x_i}\right) \begin{pmatrix} \Psi_{qqq} \\ \Psi_{qqqg} \\ \vdots \end{pmatrix} = \begin{pmatrix} \langle qqg|H_I|qqg\rangle & \langle qqg|H_I|qqqg\rangle & \cdots \\ \langle qqgq|H_I|qqg\rangle & \cdots & \\ \vdots & & \end{pmatrix} \begin{pmatrix} \Psi_{qqq} \\ \Psi_{qqqg} \\ \vdots \end{pmatrix}, \quad (16)$$

where  $H_{QCD}^{LF} = H_0 + H_I$ . Note that  $\Psi_n^S(x_i, \kappa_{\perp i}, \lambda_i)$  is only a function of  $(x_i, \kappa_{\perp i})$  as a result of the kinematic boost symmetry in light-front theory.

A complete understanding of  $g_T$  depends of course on the solution of eq.(16). For some of the approaches to solve the above bound state equation see refs. [12–14]. But here without explicitly solving the nucleon bound state from eq.(16), we show that the dominant contributions from  $g_T^{k_{\perp}}$  and  $g_T^g$  are proportional to quark mass and the transverse polarized distribution  $\Delta q_i^T(x, Q^2)$ .

From eq.(16), as we see the higher Fock states in the hadronic bound states are generated by the interaction part of QCD Hamiltonian. For large  $Q^2$ , we can rewrite the state eq.(15) as the bound state  $|\Phi(P, S, \mu)\rangle$  at hadronic scale  $\mu \sim M$  plus the radiative corrections from the high energy  $H_I$  in the  $\xi^+$ -ordering perturbative expansion:

$$|PS\rangle = \sum_{n=0}^{\infty} \left( \frac{H_I}{P^- - H_0} \right)^n |\Phi(P, S, \mu)\rangle, \quad (17)$$

where all quarks and gluons in  $H_I$  are restricted to  $\mu^2 \leq \kappa_{\perp}^2 \leq Q^2$ . Then,

$$g_T(x, Q^2) = \frac{1}{8\pi(S_{\perp} - \frac{P_{\perp}}{P^+}S^+)} \int_{-\infty}^{\infty} d\eta e^{-i\eta x} \sum_{n_1, n_2} \langle P, S, \mu | n_1 \rangle \langle n_1 | \sum_{n=0}^{\infty} \left( \frac{H_I}{P^- - H_0} \right)^n \times \left\{ (O_m + O_{k_{\perp}} + O_g) + h.c \right\} \sum_{n'=0}^{\infty} \left( \frac{H_I}{P^- - H_0} \right)^{n'} | n_2 \rangle \langle n_2 | P, S, \mu \rangle, \quad (18)$$

where  $|n\rangle$  is a simple notation of  $|n, x_i P^+, x_i P_{\perp} + k_{\perp i}, \lambda_i\rangle$ .

We first consider those terms in eq.(18) with  $|n_1\rangle = |n_2\rangle$ . This will immediately lead to  $g_T \sim \Delta q_i^T(x, Q^2)$  for large  $Q^2$ , and the coefficient is determined by the matrix element

$$\langle n_1 | \sum_{n=0}^{\infty} \left( \frac{H_I}{P^- - H_0} \right)^n (O_m + O_{k_{\perp}} + O_g) \sum_{n'=0}^{\infty} \left( \frac{H_I}{P^- - H_0} \right)^{n'} | n_1 \rangle \xrightarrow{\text{large } Q^2} \langle 1 | \sum_{n=0}^{\infty} \left( \frac{H_I}{P^- - H_0} \right)^n (O_m + O_{k_{\perp}} + O_g) \sum_{n'=0}^{\infty} \left( \frac{H_I}{P^- - H_0} \right)^{n'} | 1 \rangle, \quad (19)$$

here we denoted  $|1\rangle = |y, k_\perp, s_\perp\rangle$  which means that we have suppressed the states of all the spectators, while  $y = k^+/P^+$ .

Without the QCD correction, it is easy to show that only the quark mass term contributes to  $g_T$  in eq.(19),

$$M_T^m(x, y) = e_q^2 m_q \delta(y - x), \quad M_T^{k_\perp}(x, y) = 0 = M_T^g(x, y), \quad (20)$$

where  $M_T^i \equiv \frac{1}{4\pi} \int_{-\infty}^{\infty} d\eta e^{-i\eta x} \langle 1 | O_i | 1 \rangle$ . The physical picture of this result is as follows. In terms of the helicity basis eq.(14),  $g_T$  measures helicity flip of quarks. The quark mass term  $O_m$  already flips the helicity of one quark so that its matrix element in eq.(19) does not vanish. But the operator  $O_{k_\perp}$  and  $O_g$  do not change quark helicity of the states, the corresponding matrix elements must vanish.

Next, we consider the QCD corrections up to order  $\alpha_s$ . We find that all the three matrix elements in eq.(19) have the nonzero contribution to  $g_T$ ,

$$M_T^m(y, x, Q^2) = e_q^2 m_q^R \left\{ \delta(y - x) + \frac{\alpha_s}{2\pi} C_f \ln \frac{Q^2}{\mu^2} \left[ \frac{2}{y - x} - \delta(y - x) \left( \frac{3}{2} + \int_0^1 dx' \frac{1 + x'^2}{1 - x'} \right) \right] \right\}, \quad (21)$$

$$M_T^{k_\perp}(y, x, Q^2) = -e_q^2 m_q^R \frac{\alpha_s}{2\pi} C_f \ln \frac{Q^2}{\mu^2} \frac{(y - x)}{y^2}, \quad (22)$$

$$M_T^g(y, x, Q^2) = e_q^2 m_q^R \frac{\alpha_s}{2\pi} C_f \ln \frac{Q^2}{\mu^2} \frac{\delta(y - x)}{2}, \quad (23)$$

where  $\mu^2 > (m_q^R)^2$ , and  $m_q^R$  is the renormalized mass at the hadronic scale  $\mu$ . [The term  $\sim \frac{3}{2}\delta(y - x)$  in eq.(21) is a result of replacing the bare quark mass by the renormalized one. Note that missing this mass renormalization effect will lead to the violation of Burkhardt-Cottingham sum rule]. It shows that up to order  $\alpha_s$ , the matrix elements from  $O_{k_\perp}$  and  $O_g$  in eq.(19) are also proportional to the quark mass and they do provide a similar contribution to  $g_T(x, Q^2)$  as that of  $O_m$ .

What is the physical reason that makes the matrix elements of  $O_{k_\perp}$  and  $O_g$  have nonzero contribution to  $\Delta q_i^T(x)$  when the QCD correction is considered? The answer comes from the underlying QCD dynamics on the light-front. When QCD is quantized on the light-front,

one can find that there is a quark-gluon interaction term in the QCD Hamiltonian which is proportional to quark mass (see ref. [10]),

$$-gm_q\psi_+^\dagger\left(\not{A}_\perp\frac{1}{i\partial^+}+\frac{1}{i\partial^+}\not{A}_\perp\right)\psi_+. \quad (24)$$

At the canonical level, only this term can flip quark helicities in QCD. The nonzero contributions of  $g_T^{k_\perp}$  and  $g_T^g$  arise because the matrix element of eq.(19) contains the helicity flip from this mass term in  $H_I$ . Therefore, it is this helicity flip interaction of QCD that generates the contributions from  $g_T^{k_\perp}$  and  $g_T^g$  that is proportional to  $m_q^R$  and  $\Delta q_i^T(x, Q^2)$ .

Meanwhile, the matrix elements of  $O_{k_\perp}$  and  $O_g$  in eq.(18) also have the contributions to  $g_T$  that are not proportional to the transverse polarized distribution. These correspond to the cases where i) although  $n_1 = n_2$  the single quark states of the matrix element in eq.(19) are transversely polarized in the opposite direction, and ii)  $n_1 \neq n_2$  (different by a gluon). The corresponding contributions to  $g_T$  are proportional to the non-diagonal matrix elements given by  $\Delta\mathcal{K}_i$  and  $\mathcal{T}_i$  in eq.(10), respectively. In other words,  $\Delta\mathcal{K}_i$  and  $\mathcal{T}_i$  only contain the part of the contributions from  $g_T^{k_\perp}$  and  $g_T^g$  that does not have the simple parton picture.

Now, as we see the first term in eq.(10) that is proportional to quark mass contains all the contributions from the three terms in eq.(5) after we replace the bare quark mass by the renormalized one, where the contributions from  $g_T^{k_\perp}$  and  $g_T^g$  originate from the helicity flip quark-gluon interaction in QCD. It is well known that on the light-front the helicity is just the chirality. Helicity flip corresponds to chiral symmetry breaking on the light-front. Thus, only the helicity flip interactions, such as the one given by eq.(24), are responsible for the chiral symmetry breaking in light-front QCD. As it has been pointed out [14], the light-front QCD vacuum can be simplified in a cutoff theory so that the dynamics of the spontaneous chiral symmetry breaking in nonperturbative QCD can become an explicit chiral symmetry breaking by the manifestation of effective quark-gluon interactions in the QCD Hamiltonian. Any such interaction that is responsible for the spontaneous chiral symmetry breaking must be a helicity flip interaction. These interactions can contribute to  $g_T$  just in the same way as the canonical interaction of eq.(24). Thus, there is a contribution to  $g_T$  that arises from



the spontaneous chiral symmetry breaking in nonperturbative QCD. This contribution can be simply taken into account by requiring that the renormalized quark mass parameter does not vanish in the chiral limit. Therefore, we can effectively write  $m_q^R = m_q^c + \chi_q$ , where  $m_q^c$  is a current quark mass and  $\chi_q$  is associated with the spontaneous chiral symmetry breaking in QCD.

Meanwhile, the transverse polarized distribution  $\Delta g_i^T(x)$  which has the probabilistic interpretation is proportional to the modulus squared of the amplitudes of all the Fock states in eq.(15). But  $\Delta \mathcal{K}_i$  and  $\mathcal{T}_i$  are the off-diagonal matrix elements that are proportional to the amplitude mixings with different Fock states. These are smaller in comparison to the modulus squared of amplitudes and also have potential cancellations between different terms due to the orthogonality of different Fock states.

As a result, the terms proportional to  $\Delta \mathcal{K}_i$  and  $\mathcal{T}_i$  in eq.(10) should be much smaller than the contribution from  $\Delta q_i^T$ , and can be reasonably neglected. Therefore,  $g_T(x, Q^2)$  can be simply reduced to

$$g_T(x, Q^2) = \sum_i e_i^2 \frac{m_i^c + \chi_i}{2xM} \Delta q_i^T(x, Q^2), \quad (25)$$

where up to the leading  $\log Q^2$  QCD corrections,

$$\Delta q_i^T(x, Q^2) = \Delta q_i^T(x, \mu^2) + \frac{\alpha_s}{2\pi} C_f \ln \frac{Q^2}{\mu^2} \int_x^1 \frac{dy}{y} P_{qq}^T\left(\frac{x}{y}\right) \Delta q_i^T(y, \mu^2) \quad (26)$$

with

$$P_{qq}^T(x) = \frac{1 + 2x - x^2}{(1 - x)_+} + \frac{1}{2} \delta(1 - x), \quad (27)$$

which is obtained from eq.(21-23).

The physical picture of  $g_T$  is clear now. It probes the helicity flip interactions in hadrons on the light-front. Its dominant part is proportional to the transverse polarized parton distribution so that it has the well-defined parton picture. Since parton distributions are manifestation of the nonperturbative QCD dynamics and helicity flip on the light-front describes chiral symmetry breaking, the structure function  $g_T$  indeed directly measures the

QCD dynamics of chiral symmetry breaking. We can determine this chiral symmetry breaking effect in  $g_T$  by introducing the parameter  $\chi_i$  which is of the order  $\Lambda_{QCD}$ . This physical picture is extracted from the dominant contributions of the quark-gluon interactions by analyzing the hadronic state in terms of Fock space wavefunctions on the light front. Such an analysis is extremely difficult to perform in the standard operator product expansion method.

To examine this picture, we shall next compute  $g_2$ . By directly calculating  $g_1(x, Q^2)$  up to the leading  $\log Q^2$ , we have

$$g_1(x, Q^2) = \sum_i \frac{e_i^2}{2} \Delta q_i^L(x, Q^2), \quad (28)$$

where  $\Delta q_i^L(x, Q^2)$  satisfies the same form of eq.(26) but  $P_{qq}^T(x)$  is replaced by

$$P_{qq}(x) = \frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x). \quad (29)$$

In both eqs.(25) and (28), we have not included the possible contributions from polarized gluon distributions.

We can now extract  $g_2$  from our results of  $g_1$  and  $g_T$ ,

$$g_2(x, Q^2) = \sum_i \frac{e_i^2}{2} \left\{ \frac{m_i^c + \chi_i}{xM} \Delta q_i^T(x, Q^2) - \Delta q_i^L(x, Q^2) \right\}. \quad (30)$$

Although  $\Delta q_i^T(x, Q^2)$  may not be the same as  $\Delta q_i^L(x, Q^2)$  since their scale evolution functions are different [see eqs.(27) and (29)], if we would approximately take  $\Delta q_i^T(x, Q^2) \simeq \Delta q_i^L(x, Q^2)$ , then we have

$$xg_2(x, Q^2) \simeq \left( \frac{\chi}{M} - x \right) g_1(x, Q^2). \quad (31)$$

Here, we have ignored the current quark mass and taken  $\chi$  the average value of the  $u$  and  $d$  quarks. Eq.(31) is just our *oversimplified estimate* for  $g_2$ . We should also emphasize that the above result has nothing to do with the Wandzura-Wilzeck relation. Taking approximately  $\chi \simeq 200$  MeV, then  $\frac{\chi}{M} \simeq \frac{1}{5}$ . Since  $g_1$  has been accurately measured [15], we can estimate  $g_2$  from the above equation, and find that the result agrees very well with the current experimental data of  $g_2$ , as shown in Fig.1.

In conclusion, we have explored the transverse polarized structure function in DIS in terms of QCD and the hadronic bound state structure on the light-front. We find that the dominant contributions to transverse polarized structure function  $g_T$  from all the sources, the quark mass, the transverse quark momentum and the quark-gluon coupling, originate from the chiral symmetry breaking interactions in light-front QCD, and they are proportional to transverse polarized parton distribution. The interference effects from the transverse quark momentum and the quark-gluon coupling in eq.(10) are less important at high  $Q^2$ . As a result of the nonperturbative QCD dynamics of chiral symmetry breaking, we would expect that the magnitude of  $g_T$  is close to that of  $g_1$  at high  $Q^2$ , namely,  $g_2 = g_T - g_1$  is very small. If the chiral symmetry breaking would not play the dominant role in  $g_T$ , one would have a small value for  $g_T$  so that  $g_2$  would be close to  $-g_1$ . Thus, further experimental measurements of  $g_T$  at high  $Q^2$  can provide a precise test of the relation between  $g_T$  and the dynamical chiral symmetry breaking proposed in this work.

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## FIGURES

FIG. 1. The prediction is extracted from the  $g_1^p$  data [15] using eq.(31). The  $g_2^p$  data is from SLAC E143 [1].

